Ζήτημα 1 ΟΜΑΔΑ Α' Ιαν. 2014

$$z^4 = i = 1 \measuredangle \frac{\pi}{2} \Rightarrow z = \sqrt[4]{1}\ \measuredangle \left(\frac{1}{4}\frac{\pi}{2} + \frac{2\pi}{4}k\right) \qquad k=0,1,2,3$$

Άρα

$k=0 \Rightarrow z = 1 \measuredangle \frac{\pi}{8}$ $\qquad k=2 \Rightarrow z = 1 \measuredangle \frac{9\pi}{8}$

$k=1 \Rightarrow z = 1 \measuredangle \frac{5\pi}{8}$ $\qquad k=3 \Rightarrow z = 1 \measuredangle \frac{13\pi}{8}$ •

Ζήτημα 2

$$\begin{vmatrix} 5 & -2 & 4 & -1 \\ 0 & 1 & 5 & 2 \\ 1 & 2 & 0 & 1 \\ -3 & 1 & -1 & 1 \end{vmatrix} = 5\begin{vmatrix} 1 & 5 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & 4 & -1 \\ 1 & 5 & 2 \\ 1 & -1 & 1 \end{vmatrix} + 3\begin{vmatrix} -2 & 4 & -1 \\ 1 & 5 & 2 \\ 2 & 0 & 1 \end{vmatrix} =$$

$$= 5(5-4+1-10) + 1(-10+8+1+5-4-4) + 3(-10+16+10-4) = -8$$ •

Ζήτημα 3

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 2 & 1 & 5 & 2 \\ 3 & 4 & 3 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -5 & 9 & 0 \\ 0 & -5 & 9 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -5 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} \times 5 \\ \times 3 \end{array} \sim \left(\begin{array}{ccc|c} 5 & 15 & -10 & 5 \\ 0 & 15 & -27 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 5 & 0 & 17 & 5 \\ 0 & 15 & -27 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{17}{5} & 1 \\ 0 & 1 & -\frac{27}{15} & 0 \end{array}\right)$$

Δηλ. $x_1 + \frac{17}{5}x_3 = 1 \Leftrightarrow x_1 = 1 - \frac{17}{5}x_3$

$x_2 - \frac{27}{15}x_3 = 0 \Leftrightarrow x_2 = \frac{9}{5}x_3$

(μονοπαραμετρική απειρία λύσεων) •

Ζήτημα 4

$$\lim_{x\to\infty} \frac{\ln(x^2+1)}{\ln(x^3+1)} = \lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{\frac{3x^2}{x^3+1}} = \lim_{x\to\infty} \frac{2x(x^3+1)}{3x^2(x^2+1)} = \frac{2}{3}$$ •

Ζήτημα 5

Ως γνωστόν $\sinh' x = \cosh x$, $\cosh' x = \sinh x$ και

$$\sinh(0) = \frac{e^0 - e^{-0}}{2} = 0 \qquad \cosh(0) = \frac{e^0 + e^{-0}}{2} = 1$$

Άρα $\sinh x = \cancel{\sinh(0)} + \frac{x-0}{1!}\cosh(0) + \frac{(x-0)^2}{2!}\cancel{\sinh(0)} + \frac{(x-0)^3}{3!}\cosh(0) + \dots =$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$ •

Ζήτημα 6

$$\int_0^1 \frac{x\,dx}{x^2+1} = \frac{1}{2}\int_0^1 \frac{d(x^2+1)}{x^2+1} = \frac{1}{2}\ln(x^2+1)\Big|_0^1 = \frac{1}{2}(\ln 2 - \ln 1) = \frac{\ln 2}{2}$$ •

Ζήτημα Bonus (για δυνατούς λύτες ...)

$$y = \operatorname{arcsinh} x \Leftrightarrow x = \sinh y \Rightarrow \frac{dx}{dy} = \cosh y = \sqrt{1+\sinh^2 y} = \sqrt{1+x^2} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$ • $\qquad$ (σημ: $\cosh^2 y - \sinh^2 y = 1$)

Ζήτημα 1 ΟΜΑΔΑ Β' Ιαν 2014

$$z^4 = -i = 1 \measuredangle -\frac{\pi}{2} \Rightarrow z = \sqrt[4]{1} \measuredangle \left(\frac{1}{4}\left(-\frac{\pi}{2}\right) + \frac{2\pi}{4} \cdot k \right) \quad k = 0,1,2,3$$

Άρα $k=0 \Rightarrow z = 1 \measuredangle -\frac{\pi}{8}$ $\quad k=2 \Rightarrow z = 1 \measuredangle \frac{7\pi}{8}$

$k=1 \Rightarrow z = 1 \measuredangle \frac{3\pi}{8}$ $\quad k=3 \Rightarrow z = 1 \measuredangle \frac{11\pi}{8}$ ●

Ζήτημα 2

$$\begin{vmatrix} 2 & 1 & 3 & -1 \\ 0 & 2 & 3 & -2 \\ 1 & -2 & 0 & 3 \\ -2 & 1 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} 2 & 3 & -2 \\ -2 & 0 & 3 \\ 1 & 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 & -1 \\ 2 & 3 & -2 \\ 1 & 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 & -1 \\ 2 & 3 & -2 \\ -2 & 0 & 3 \end{vmatrix} =$$

$$= 2(9+8-12+6) + (3-6-4+3+4-6) + 2(9+12-6+8) = 10$$ ●

Ζήτημα 3

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 3 & 1 & -1 & 2 \\ 4 & 3 & -4 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & -5 & 8 & -1 \\ 0 & -5 & 8 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 5 & -8 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{matrix} \times 5 \\ \times 2 \\ \sim \end{matrix} \left(\begin{array}{ccc|c} 5 & 10 & -15 & 5 \\ 0 & 10 & -16 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 5 & 0 & 1 & 3 \\ 0 & 5 & -8 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{3}{5} \\ 0 & 1 & -\frac{8}{5} & \frac{1}{5} \end{array}\right)$$

Δηλ. $x_1 + \frac{x_3}{5} = \frac{3}{5} \Leftrightarrow x_1 = \frac{3}{5} - \frac{x_3}{5}$

$x_2 - \frac{8}{5}x_3 = \frac{1}{5} \Leftrightarrow x_2 = \frac{1}{5} + \frac{8}{5}x_3$ ●

(μονοπαραμετρική απειρία λύσεων)

Ζήτημα 4

$$\lim_{x\to\infty} \frac{\ln(x^3+1)}{\ln(x^4+1)} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^3+1}}{\frac{4x^3}{x^4+1}} = \lim_{x\to\infty} \frac{3x^2(x^4+1)}{4x^3(x^3+1)} = \frac{3}{4}$$ ●

Ζήτημα 5 Ως γνωστόν $\cosh' x = \sinh x$, $\sinh' x = \cosh x$ και

$\sinh(0) = \frac{e^0 - e^{-0}}{2} = 0 \quad \cosh(0) = \frac{e^0 + e^{-0}}{2} = 1$

Άρα $\cosh x = \cosh(0) + \frac{x-0}{1!}\cancel{\sinh(0)} + \frac{(x-0)^2}{2!}\cosh(0) + \frac{(x-0)^3}{3!}\cancel{\sinh(0)} + \dots$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

Ζήτημα 6

$$\int_0^1 \frac{x^2 dx}{x^3+1} = \frac{1}{3}\int_0^1 \frac{d(x^3+1)}{x^3+1} = \frac{1}{3} \ln(x^3+1)\Big|_0^1 = \frac{1}{3}(\ln 2 - \ln 1) = \frac{\ln 2}{3}$$ ●

Ζήτημα Bonus (για δυνατούς λύτες...)

$y = \operatorname{arccosh} x \Leftrightarrow x = \cosh y \Rightarrow \frac{dx}{dy} = \sinh y = \sqrt{\cosh^2 y - 1} = \sqrt{x^2-1} \Rightarrow$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}}$ ● $\quad$ (σημ. $\cosh^2 y - \sinh^2 y = 1 \Leftrightarrow \sinh^2 y = \cosh^2 y - 1$)